



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 58

Stock Index Futures – 2

J P Singh

Department of Management Studies



STOCK PICKING

- **An investor might be very uncertain about the performance of the market as a whole, but confident that some stocks in his long portfolio will outperform the market.**



STRATEGY

- A **short** position using index futures can be used to reduce the beta of the portfolio and thereby immunize the investor from the risk arising from market moves.
- It leaves the hedger exposed only to the intrinsic performance of the portfolio.

EXAMPLE

- Consider an investor who in April holds 20,000 IBM shares, each worth \$100. The investor feels that the market will be very volatile over the next three months but that IBM has a good chance of outperforming the market. The investor decides to use the August futures contract on the S&P 500 to hedge the market's return during the three month period. The beta of IBM is estimated at 1.1.



- Suppose that the current futures price for the August contract on the S&P 500 is 900. Each contract is for delivery of \$250 times the index.
- The investor closes out the position in July. Suppose IBM falls to \$90 and the futures price of the S&P 500 falls to 750.
- Calculate the net profit/loss on the hedged portfolio.



SOLUTION 1

- No of futures contracts shorted: $\beta V_s / z V_f^*$ **Why?**
- $= 1.10 * 20,000 * 100 / (250 * 900) = 9.78 = 10$ (rounded off)
- Net Loss on stock $= 20,000 * (90 - 100) = 200,000$
- Profit on futures $= 10 * 250 * (900 - 750) = 375,000$
- Net profit on hedged portfolio $= 175,000$ due to stock picking.
- % loss on market $150 / 900 = 16.67\%$
- Beta of stock $= 1.1$
- Hence, loss due to systematic risk on his IBM portfolio $= 1.1 * 16.67\% = 18.33\% = 366,667$.



- Net loss on shares: 200,000
- Loss due to systematic risk: 366,667
- Therefore, profit due to unsys risk: 166,667
- Profit due to futures: 375,000
- Profit due to futures annuls loss due to sys risk.
Hence, profit due to unsystematic risk remains.



- The superior return of the IBM stock could be attributed to some special knowledge, expertise available to the investor or access to some inside information at investment.
- Since the market would not be aware of such information etc at $t=0$, it would not have been built into the price at the time of investment and hence, the under pricing.
- Further, market will view this superior return as random return since the market did not have any information.



$$\Delta \tilde{I} = Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}$$

BETA OF HEDGED PORTFOLIO

$$\tilde{R}_I = \frac{\Delta \tilde{I}}{I_0} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{I_0} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{I_{S,0}} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{Q_S \times S_0}$$

(since initial investment in futures is zero i.e. $I_{F,0} = 0$)

$$= \frac{\Delta \tilde{S}}{S_0} - \frac{Q_F}{Q_S} \frac{F_0}{S_0} \frac{\Delta \tilde{F}}{F_0} = \tilde{R}_S - \frac{Q_F F_0}{I_{S,0}} \tilde{R}_F = \tilde{R}_S - \frac{V_{F,0}}{I_{S,0}} \tilde{R}_F$$



Now, $E(\tilde{R}_S) = R_f + \beta_S (E(\tilde{R}_m) - R_f)$.

Similarly, $E(\tilde{R}_F) = \beta_F (E(\tilde{R}_m) - R_f)$

The riskfree rate does not appear because futures do not involve any upfront investment so there is no opportunity cost for funds tied up in futures.

Also $E(\tilde{R}_I) = R_f + \beta_H (E(\tilde{R}_m) - R_f)$

$$\beta_H = \beta_S - \frac{V_{F,0}}{I_{S,0}} \beta_F. \quad \text{For complete hedging}$$

$$\beta_H = 0; V_{F,0} = \beta_S \frac{I_{S,0}}{\beta_F}; N = \frac{V_{F,0}}{V_{F,0}^*} = \beta_S \frac{I_{S,0}}{\beta_F V_{F,0}^*}$$

$$\text{For index futures } \beta_F = 1 \text{ so that } N = \beta_S \frac{I_{S,0}}{V_{F,0}^*}$$

EXAMPLE 2

- A portfolio manager has a portfolio of long positions in stocks valued at USD 1.5m with a beta of 1.50 at $t=0$ when the S & P 500 index is 1500. He desires to hedge this portfolio for a period of 3 months using S& P 500 index futures with a maturity of four months. The index has a continuously compounded yield of 4% and the riskfree rate is 10.00% cont. compd.. The contract multiple is USD 250. Calculate the number of contracts required. If the index actually falls by 10% at $t=3$ months from its original value assess the performance of the hedged portfolio. **Use no arbitrage pricing wherever required.**



CALCULATION OF NO OF CONTRACTS		
SPOT VALUE OF INDEX	1500	
RISK FREE RATE	10%	r
YIELD ON INDEX	4%	q
MATURITY	4	MONTHS
NO ARBITRAGE FUTURE VALUE OF INDEX	1530.302	$\text{EXP}[(r-q)T]$
CONTRACT MULTIPLE	250	z
NO ARBITRAGE PRICE OF FUTURES CONTRACT	382575.5	$V(f,0)$
EXPOSURE	1500000	$V(s)$
BETA OF EXPOSURE	1.5	β
NO OF CONTRACTS	5.881192	$\beta V(s)/V(f,0)$
ROUNDED OFF	6	

EVALUATION OF HEDGE		
HEDGE PERIOD	3	MONTHS
DECLINE IN INDEX AT END OF HEDGE PERIOD	10%	
VALUE OF INDEX AT END OF HEDGE PERIOD	1350	-
RISK FREE RATE	10%	r
YIELD ON INDEX	4%	q
MATURITY	1	MONTHS
NO ARBITRAGE FUTURE VALUE OF INDEX	1356.767	$\text{EXP}[(r-q)T]$
CONTRACT MULTIPLE	250	z
NO ARBITRAGE PRICE OF FUTURES CONTRACT	339191.7	$V(f,0)$
PROFIT ON SHORT FUTURES	260302.7	

LOSS ON SPOT	
FALL IN INDEX	10%
YIELD ON INDEX FOR 3 MONTHS	0.01
MARKET RETURN	-9%
RISK FREE RATE FOR 3 MONTHS	0.025
BETA	1.5
RETURN ON PORTFOLIO USING CAPM	-0.1475
LOSS IN VALUE OF PORTFOLIO	-221250
NET GAIN ON HEDGED PORTFOLIO	39052.66
RETURN ON HEDGED PORTFOLIO	0.026035



INDEX ARBITRAGE: NO ARBITRAGE FUTURES PRICES

$$F = S[1 + (r - \delta)T] \quad (\text{using simple rates})$$

$$F = S[1 + (r - \delta)/q]^{qT} \quad (\text{using discrete compounded rate})$$

$$F = Se^{(r_c - \delta_c)T} \quad (\text{using continuously compounded rates})$$



STOCK INDEX ARBITRAGE

- **Stock Index Arbitrage is an attempt to exploit any futures mispricing relative to the index level. When futures prices lie outside their no-arbitrage bounds, arbitrageurs will quickly act to realize the (near) riskless profits by buying cheap stock and selling futures, or buying cheap futures and selling stock.**



ARBITRAGE BOUNDS

- If $F > S + CC - CR$, then arbitrageurs' achieve arbitrage profits by purchases of stock. This will increase stock prices. Their sales of futures will depress futures prices, until equilibrium is again reached ($F \cong S + CC - CR$), and no arbitrage opportunities exist.
- This is CASH & CARRY arbitrage.



- Similarly, if $F < S + CC - CR$, arbitrage involves the buying of cheap futures and the sale of expensive stock. Their purchases of futures will increase futures prices, until equilibrium is again reached ($F \cong S + CC - CR$), and no arbitrage opportunities exist.
- This is **REVERSE CASH & CARRY** arbitrage.



ASSUMPTIONS UNDERLYING INDEX ARBITRAGE

- We can purchase or short sell 'actual stocks' to 'replicate' the performance of the stock index and this can be achieved at low transaction cost.
- The actual dividends paid out from the 'replication' stock portfolio can be approximated by the constant (value weighted) dividend yield (per unit of time).
- Discrete dividend payments from our 'replication' stock portfolio are known with certainty.



BASIS RISK IN ARBITRAGE

- Traders only buy (or sell) a proportion of the stocks in the S&P500 say, so their cash position might not exactly mirror that in the futures index.
- There are transactions costs
- Discreteness of contracts
- Timing of dividend payment may be uncertain.
- It might not be possible to do the trades in futures and the stock market at exactly the same time, and in the meantime the prices of some of the underlying stocks might change.



EXAMPLE 5

- Suppose the current index S_0 is 200, riskfree rate (r) is 10% continuously compounded, yield (q) is 3% also cont. compd. and the maturity of index futures (T) is 4 months. However, the quoted futures price is $F = 210$. Examine whether there is any scope for index arbitrage.



SOLUTION 5

- The ‘synthetic’ or ‘fair’ futures price is $F_0 = S_0 \exp(r-q)T = 204.72$. However, if the quoted futures price is $F = 210$ then an arbitrage opportunity is possible.



COST OF INDEX	200
RISKFREE RATE	10%
YIELD	3%
MATURITY	0.333333
AMT TO BE BORROWED	198.01
AMT TO BE REPAYED	204.7215
ACTUAL FORWARD PRICE	210
PROFIT	5.278463



- You sell the futures contract today at $F = 210$, borrow 198.01 @ $r = 10\%$ for 4 months, buy 0.99 of the underlying index stocks today for 198.01. Due to the 3% yield, this stock portfolio will grow to one unit in four months. You owe 204.72 after 4 months against your borrowing. You deliver the stocks against the short futures position after 4 months and receive $F = 210$ (from the long in the futures), making an arbitrage profit of 5.28 ($= 210 - 204.72$).

